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QUASI-PERIODIC ORBITS IN THE VICINITY OF THE SUN-EARTH SYSTEM L_2 POINT AND THEIR IMPLEMENTATION IN “SPECTR-RG” AND “MILLIMETRON” MISSIONS

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This work considers construction of quasi periodic orbits in the vicinity of the Sun-Earth system L_2 libration point for the upcoming Roscosmos “Spectr-RG” and “Millimetron” missions. The problem is considered in the full dynamical model, the initial approximation is built with the help of Richardson technique extended on Elliptic Restricted Three-body Problem case. Selection of A_x and A_z oscillation amplitudes provides quasi periodic orbits of different types (halo or Lissajous orbits) and geometries. Transfer trajectories are selected on the L_2 point stable manifold with the help of parameter extension method. Loose control stationkeeping strategy provides up to 10 years of spacecraft operation in the selected orbit with given geometry. The obtained halo orbits and transfer trajectories have been selected as the nominal trajectories for “Spectr-RG” and “Millimetron” missions.

I. INTRODUCTION

Quasi periodic orbits in the vicinity of collinear libration points L_1 and L_2 have been widely used for deployment of a number of NASA and ESA spacecraft carrying out astrophysical studies. These orbits are favourable as they provide stable Sun-Earth-spacecraft configuration, space telescope placed in such orbit can maintain its orientation relatively to Sun and Earth. Space observatory has great advantage over a ground based station as it has no atmosphere shield which means no dependence on weather and much higher sensibility. Due to these facts Russian federal space agency Roskosmos has planned two missions going to the L_2 point for the next few years: “Spectr-RG” spacecraft is intended to be placed in a compact Lissajous orbit in the vicinity the L_2 point in 2016; on the opposite “Millimetron” spacecraft should be going out far from the ecliptics plane using a large radius halo orbit, launch is scheduled for the end of 2018. Both spacecraft are intended to operate during the 7 years period. To keep the spacecraft in the intended quasi periodic orbit some stationkeeping strategy should be applied. According to projects' requirements total stationkeeping costs for this period must not overcome 200 m/sec.

II. COLLINEAR LIBRATION POINTS' DYNAMICS

Libration points' dynamics is usually studied within the Circular Restricted Three Body Problem (CRTBP) framework. The best description of libration points' dynamics is given in[1]: “collinear libration points are of center × center × saddle type due to the eigenvalues of the Jacobian matrix of the CRTBP vector field in these points being $\{\pm i\omega_1, \pm i\omega_2, \pm\lambda\}$. Due to the center × center part, and according to Lyapunov's center theorem, each collinear equilibrium point gives rise to two one-parameter families of periodic orbits, known as the planar and the vertical Lyapunov families of periodic orbits (Fig. 1). In addition, in each energy level close to the one of the equilibrium point, there is a two-parameter family of 2D tori, known as Lissajous orbits, that connects the two Lyapunov families. Some of these tori are foliated by periodic orbits, but most of them carry an irrational flow. Thus, considering all the energy levels, there are 4D center (neutrally stable) manifolds around these points. For a given energy level, they are just 3D sets where the dynamics has a neutral behavior.

Along the families of Lyapunov periodic orbits, as the energy increases, the linear stability of the orbits change and there appear bifurcating orbits where other families of periodic orbits appear. The first family bifurcating from the planar Lyapunov one corresponds to 3-dimensional periodic orbits symmetric with respect the $y = 0$ plane, the so-called halo orbits. At the bifurcation, two families of orbits are born, known as the Northern and Southern halo families.

Saddle part gives rise to invariant manifolds, providing natural transfer trajectories to periodic orbits.”

This work combines dynamical systems approach with some numerical techniques.

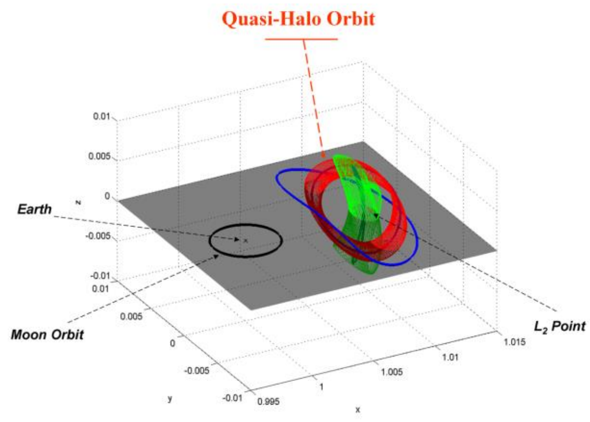

Fig.1: Periodic and quasi periodic orbits in the vicinity of the L_2 point. Blue colour represents planar and vertical Lyapunov orbits, black colour – halo orbit with red quasi periodic orbit around it, green colour – Lissajous orbit.

III. PERIODIC ORBIT APPROXIMATIONS IN CRTBP AND ERTBP

The simplest approximation to a quasi periodic orbit in the vicinity the L_2 point is the solution of the motion equations of circular restricted three body problem (CRTBP) linearized in small area around libration point[2].

$$\begin{aligned} \xi_1 &= A(t)\cos\left(\omega_1 t+\varphi_1(t)\right)+C(t)e^{\lambda t}+D(t)e^{-\lambda t} \\ \xi_2 &= -k_2 A(t)\sin\left(\omega_1 t+\varphi_1(t)\right)+k_1\left(C(t)e^{\lambda t}-D(t)e^{-\lambda t}\right) \\ \xi_3 &= B(t)\cos\left(\omega_2 t+\varphi_2(t)\right) \end{aligned} \quad [1]$$

Here *A(t)* and *B(t)* are X-Y and Y-Z plane oscillation amplitudes, average values of *A(t)* and *B(t)* coefficients are chosen at the orbit design stage, they characterize it's geometrical size in the ecliptics plane and in the plane which is orthogonal to it. *C(t)* value should be close to zero in order to prevent solution from exponential growth. *D(t)* is chosen in such a way that when *t* is equal to zero the spacecraft's motion trajectory should cross the border of the Earth's incidence sphere. In the restricted three body problem the *A*, *B*, *C*, *D* coefficients do not depend on time. We shall use this model and the coefficients to describe geometry and stability of the obtained quasi periodic orbits. It is more informative to handle dimensionless values obtained by such normalization:

$$\theta_A = \frac{A}{R_L}, \theta_B = \frac{B}{R_L}, \theta_C = \frac{C}{R_L} \quad [2]$$

R_L is distance from the L_2 point to Earth.

Another way to build more precise approximation to the CRTBP periodic solution is Richardson's 3d order approximation obtained with the help of Linstedt-Poincaré technique applied to Legendre polynomial expansion of the classical CRTBP equations of motion[3].

$$\begin{aligned} x &= a_{21}A_x^2 + a_{22}A_z^2 - A_x\cos\tau_1 + (a_{23}A_x^2 - a_{24}A_z^2)\cos 2\tau_1 + \\ &+ (a_{31}A_x^3 - a_{32}A_xA_z^2)\cos 3\tau_1 \\ y &= kA_x\sin\tau_1 + (b_{21}A_x^2 - b_{22}A_z^2)\sin 2\tau_1 + \\ &+ (b_{31}A_x^3 - b_{32}A_xA_z^2)\sin 3\tau_1 \\ z &= \delta_n A_z\cos\tau_1 + \delta_n d_{21}A_xA_z(\cos 2\tau_1 - 3) + \\ &+ \delta_n(d_{32}A_zA_x^2 - d_{31}A_z^3)\cos 3\tau_1 \end{aligned} \quad [3]$$

Where τ_1 is dimensionless time, A_x and A_z are oscillation amplitudes and there is restriction put upon A_x minimum value

$$A_{x\min} \geq \sqrt{|\Delta / l_1|} \quad [4]$$

A_z value is derived from this amplitude bounding equation

$$l_1A_x^2 + l_2A_z^2 + \Delta = 0 \quad [5]$$

And all a_{ij}, b_{ij} and d_{ij} values are constants.

$$\delta_n = 2 - n, \; n = 1,3 \quad [6]$$

Details concerning technique providing these equations are discussed in[3].

The next step taken was to move this solution from CRTBP to the Elliptic Restricted Three Body Problem (ERTBP) in order to obtain more exact approximation of periodic orbit. Transfer to elliptic problem is performed the following way: first we convert true anomaly *f* describing ERTBP evolution into dimensionless time *t* with the help of Kepler's equation.

$$\mathrm{tg}\left(\frac{E}{2}\right) = \frac{\mathrm{tg}\left(\frac{f}{2}\right)}{\sqrt{\frac{1+e}{1-e}}} \quad [7]$$

$$\begin{aligned} M &= E - e\sin E \\ t_{\text{dimensionless}} &= M \end{aligned} \quad [8]$$

Then we apply Richardson procedure, obtaining CRTBP initial approximation state vector $(x, y, z, \dot{x}, \dot{y}, \dot{z})$. After that the state vector is converted to non-dimensional Nechvil variables, depending on true anomaly and eccentricity instead of time $(\xi, \eta, \zeta, \dot{\xi}, \dot{\eta}, \dot{\zeta})$.

$$x = \rho\xi$$
$$y = \rho\eta \qquad [9]$$
$$z = \rho\zeta$$

$$\frac{dx}{df} = \frac{dx}{dt}\cdot\frac{dt}{df} \qquad [10]$$

$$\frac{dt}{df} = \frac{p^{3/2}}{\left(1+e\cos f\right)^2} \qquad [11]$$

$$\xi' = \dot{x}\frac{p^{3/2}}{1+e\cos f} + x(-e\sin f)$$
$$\eta' = \dot{y}\frac{p^{3/2}}{1+e\cos f} + y(-e\sin f) \qquad [12]$$
$$\zeta' = \dot{z}\frac{p^{3/2}}{1+e\cos f} + z(-e\sin f)$$

The ideas concerning generalization of CRTBP methods to the ERTBP are described in[4]. Finally equations of motion in Nechvil variables (1.17) describing ERTBP are obtained.

$$\Omega = \frac{1}{1+e\cos f}\left(\frac{\xi^2+\eta^2}{2} - \frac{1}{2}e\zeta^2\cos f + \frac{1-\mu}{\rho_1} + \frac{\mu}{\rho_2}\right)$$
$$\rho_1^2 = \left(\xi+\mu\right)^2 + \eta^2 + \zeta^2 \qquad [13]$$
$$\rho_2^2 = \left(\xi-1-\mu\right)^2 + \eta^2 + \zeta^2$$

$$\frac{\partial\Omega}{\partial\xi} = \frac{1}{1+e\cos f}\left(\xi - \frac{(1-\mu)(\xi+\mu)}{\rho_1^3} - \frac{\mu(\xi+\mu-1)}{\rho_2^3}\right)$$
$$\frac{\partial\Omega}{\partial\eta} = \frac{1}{1+e\cos f}\left(\eta - \frac{(1-\mu)\eta}{\rho_1^3} - \frac{\mu\eta}{\rho_2^3}\right) \qquad [14]$$
$$\frac{\partial\Omega}{\partial\zeta} = \frac{1}{1+e\cos f}\left(-\frac{(1-\mu)\zeta}{\rho_1^3} - \frac{\mu\zeta}{\rho_2^3} - e\zeta\cos f\right)$$

$$\frac{d^2\xi}{df^2} = \frac{\partial\Omega}{\partial\xi} + 2\frac{d\eta}{df}$$
$$\frac{d^2\eta}{df^2} = \frac{\partial\Omega}{\partial\eta} - 2\frac{d\xi}{df} \qquad [15]$$
$$\frac{d^2\zeta}{df^2} = \frac{\partial\Omega}{\partial\zeta}$$

IV. TRANSFER TRAJECTORY APPROXIMATION IN RTBP

Same as with quasi periodic orbits first an approximation of transfer trajectory in RTBP should be obtained. Since the libration point orbits around L_1 and L_2 points have strong hyperbolic character, their stable manifold is usually used for the transfer[1]. Transfer trajectory to the selected quasi periodic orbit is searched within the invariant manifold with help of the isoline method[5,6,7,8,9]. This method provides connection between periodic orbit dots (here comes periodic orbit approximation obtained in the previous section) and geocentric transfer trajectory parameters – the isoloines of transfer trajectory pericentre height function depending on periodic orbit parameters are built. The idea of isoline building method is to find trajectories coming out of periodic solution dots backwards in time that intersect with an injection orbit. This provides one-impulse transfer from low earth orbit (LEO) to the quasi periodic orbit. This method has been extended on non-direct transfers including Moon gravity assist, which is opportune as it provides ΔV needed to enter a more compact quasi periodic orbit (Fig. 2) – Y amplitude is close to $400\bullet10^3$ km instead of $800\bullet10^3$ km amplitude in case of direct transfer. The idea is the same, but the function of the pericentre height also depends on time in this case, setting time restrictions on the launch dates.

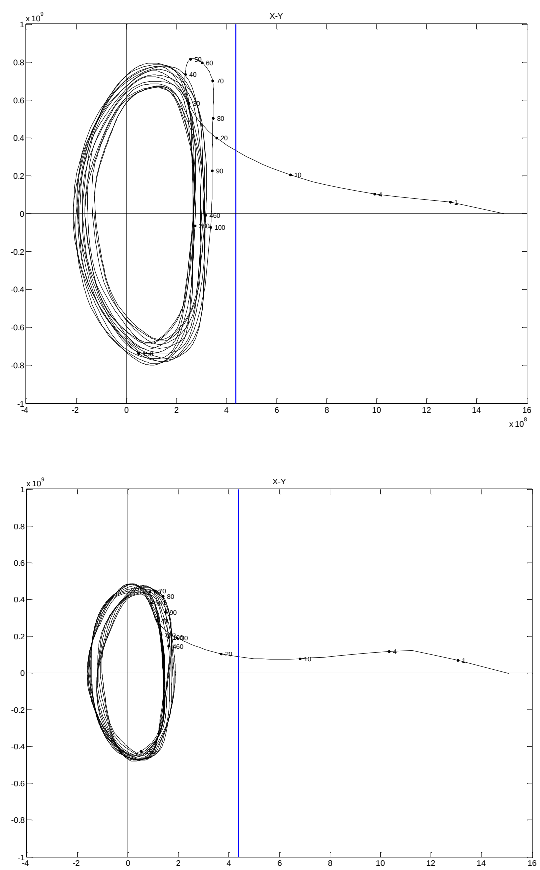

Fig.2: Top plot: Transfer trajectory without the Moon gravity assist manoeuver. Bottom plot: Transfer trajectory with the Moon gravity assist maneuver. The XY plane view in the rotating reference frame, dimension – millions of kilometers.

A decision has been made not to use Moon gravity assist manoeuver as its performance errors may affect the whole mission robustness but if we concern orbit design solely this is an opportune technique.

V. NOMINAL TRAJECTORY CALCULATION

After having obtained a number of trajectories crossing Earth-centred sphere with $r = r_{\text{E}} + h_{\text{injectionorbit}}$ within ERTBP framework the nominal transfer trajectory calculation algorithm is being applied. It has the following structure:

- The isolines built are the income data for the flight trajectory initial kinematics parameters' calculation algorithm – the initial approximation of the transfer to the halo orbit. At this stage we move the whole problem to the full numerical model used in KIAM Ballistic centre for navigational and ballistic support of currently operating missions. Launch data is selected, that defines the injection orbit, thus trajectories a sorted out.
- The initial approximation built is used for the exact calculation of the flight from the fixed LEO to the given halo orbit. The kinematics parameters' vector is counted more precisely according to the boundary conditions.

1. The velocity vector of the transfer trajectory, obtained from the initial approximation is counted more precisely according to the boundary conditions which are the given values of the orbit parameters *B* and *C = 0*.

2. The velocity vector, obtained at the stage 1 is counted more precisely according to the condition of the maximum time of the halo orbit staying in the L_2 sphere of the given radius R_{L_2}

$$R_{L_2} = r_L \sqrt{\theta_B^2 + \theta_A^2 \left(1 + k_2^2\right)} \qquad [16]$$

Here we come to the stationkeeping strategy. There are two different ways of keeping the spacecraft in desired orbit. First one is tight control strategy keeping the spacecraft in a tube around the desired periodic approximation. We have tried it, but the other way – loose control – appeared to be a more successful strategy. Every 45 days an orbit correction is performed, correction impulse vector $\Delta\vec{V}_i$ is calculated according to the condition of the maximum time of the spacecraft staying in the L_2 point vicinity of the stated radius after the correction has been implemented. The maximum time is searched for with the help of the gradient method.

$$\Delta\vec{V}_i = \frac{1}{2^q} \frac{\Delta V \max}{\left|\nabla F_c\right|} \left(\nabla F_c\right)^{\text{T}} \qquad [17]$$

F_C is the functional, describing the time while the spacecraft stays in the L_2 point vicinity of the given radius R_{L_2}.

$$F_C = t_{\text{outL2}} - t_{\text{inL2}} \qquad [18]$$

The third stationkeeping strategy is represented by another F_C expression, representing orbit geometry coefficients control.

$$F_C = \frac{1}{T} \int_{t_1}^{t_1+T} \left(\left(B(t) - \theta_B r_L \right)^2 + C(t)^2 \right) dt \qquad [19]$$

Numerical experience has proved second strategy to be more efficient. It provides quasi periodic orbits of desired geometry staying in L_2 point vicinity for 7.5 years (the intended spacecraft lifetime) with total stationkeeping costs less than 10 m/sec.

After the final trajectory has been obtained shadow zones and radio visibility zones for Russian ground stations are calculated in order to make sure that the new trajectory meets all restrictions set (Table 1). Details concerning nominal trajectories calculation algorithm are discussed in papers[7,8,9].

Figures 3 and 4 represent the obtained trajectories selected as the nominal ones for "Spectr-RG" and "Millimetron" missions. It is clear from the figures that selection of different A_x and A_z amplitudes has resulted in quite different orbit types – "Millimetron" trajectory may be classified as a quasi halo orbit while "Spectr-RG" trajectory is a pure Lissajous orbit.

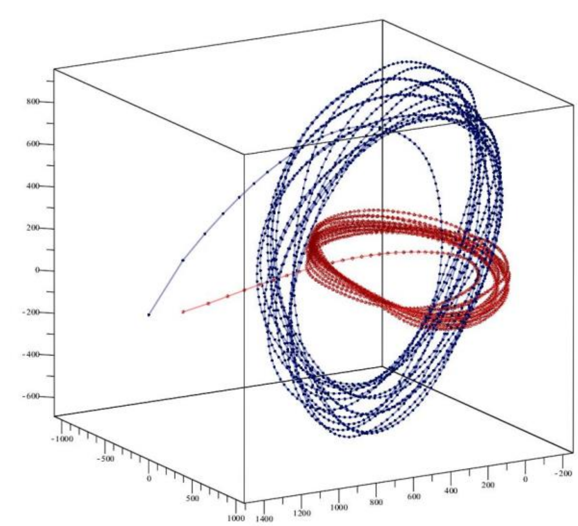

Fig. 3: 3D view of the quasi periodic orbits, proposed for "Spectr-RG" (red) and "Millimetron" (blue) spacecraft in the rotating L_2 centered reference frame (X-axis points from the Sun)

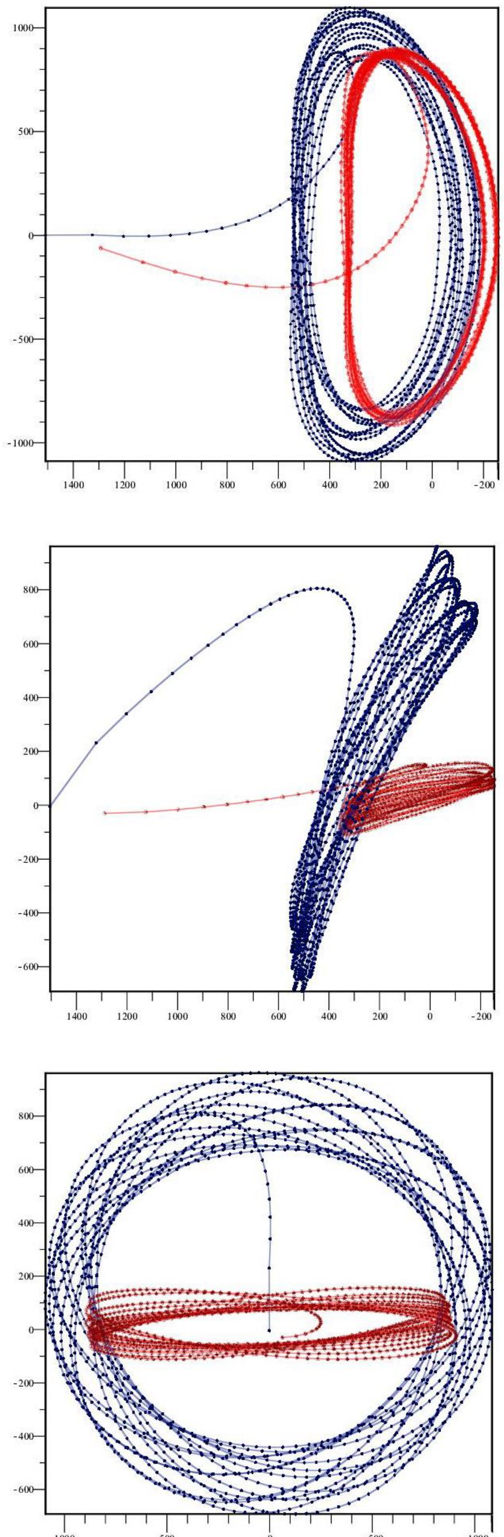

Fig. 4: 2D view of the quasi periodic orbits, proposed for "Spectr-RG" (red) and "Millimetron" (blue) spacecraft in the rotating L_2 centred reference frame (X-axis points from the Sun)

Further ballistic analysis and modelling have been performed in order to ensure missions' robustness, taking into account launcher and spacecraft engines' performance errors. Midcourse corrections (MCC) needed to correct possible Soyuz-Fregat injection dispersion and to insert into selected quasi-periodic orbit have been estimated assuming stationkeeping engine (SCE) performance errors to be 10% of magnitude and 0.5˚of direction (see Table 2). Stationkeeping ΔV costs will also increase due to engine performance and navigation errors, but will remain within given restrictions. Error values taken for this analysis represent the most pessimistic scenario, accelerometers installation on board the spacecraft provides much higher maneuver performance precision, decreasing ΔV costs more than 3 times except of the 1st MCC defined by launcher performance dispersion.

Mission	Spectr-RG		Millimetron	
Maneuver	Av, m/sec	Max, m/sec	Av, m/sec	Max, m/sec
1st MCC	22.2	43.8	22.2	43.8
2nd MCC	2.4	10.1	3.3	13.5
3d MCC	0.3	1.3	0.4	1.6
4th MCC	1.2	2.8	1.4	3.0
Stationkeeping ΔV costs	35.0	124.0	43.0	173.0
Total ΔV costs	61.1	182.0	70.3	234.9

Table 1: Midcourse corrections and stationkeeping maneuvers ΔV costs (Av-average, Max- maximum).

Launch window analysis has been carried out, the results for the expected launch dates for Spectr-RG and Millimetron mission are given in Fig. 5,6. This diagram represents all solutions obtained, not taking eclipse and radio visibility conditions into account, it just gives the idea of possible launch windows from the energetic point of view, considering stationkeeping ΔV costs dependence on date and time of spacecraft transition from the injection orbit to the transfer trajectory.

VI. CONCLUSIONS

A new method of quasi periodic orbits construction, generalizing Lindstedt-Poincaré-Richardson technique for the ERTBP case has been developed and programmed. M.L. Lidov's isoline building method providing one-impulse transfers from LEO to a quasi periodic orbit in the vicinity of a collinear libration point has been extend on gravity assist trajectory class. An algorithm calculating stationkeeping impulses for the quasi periodic orbit maintenance has been developed and programmed. It provides stationkeeping strategies for spacecraft lifetime over 7 years with total ΔV costs within

Quasi-periodic orbit parameters	Requirements/ Constraint Driver(s)	Spectr-RG	Millimetron
Orbit geometry: Y amplitude	Sp-RG: Communications Millimetron: Science	Maximum 900.000 km	Minimum 900.000 km
Orbit geometry: Z amplitude	Sp-RG: Communications Millimetron: Science	Maximum 600.000 km	Minimum 900.000 km
Maximum SCE finite-burn duration	Propulsion	1800.0 sec	2000.0 sec
Minimum precision of SCE finite-burn duration	Propulsion	0.1 sec	0.2 sec
Estimated MCC average ΔV	Mass & Propulsion	55 m/s	28 m/s
Stationkeeping available ΔV	Mass & Propulsion	228 m/s	287 m/s
Mission lifetime goal	Science	7.5 years	7.5 years
Lunar / Earth Eclipse	Power and Thermal	None allowed	None allowed
Radio visibility	Communications, navigation & control	Must be provided every day for Northern hemisphere ground stations	Southern hemisphere ground stations should be used

Table 2: Quasi-periodic orbit constraints.

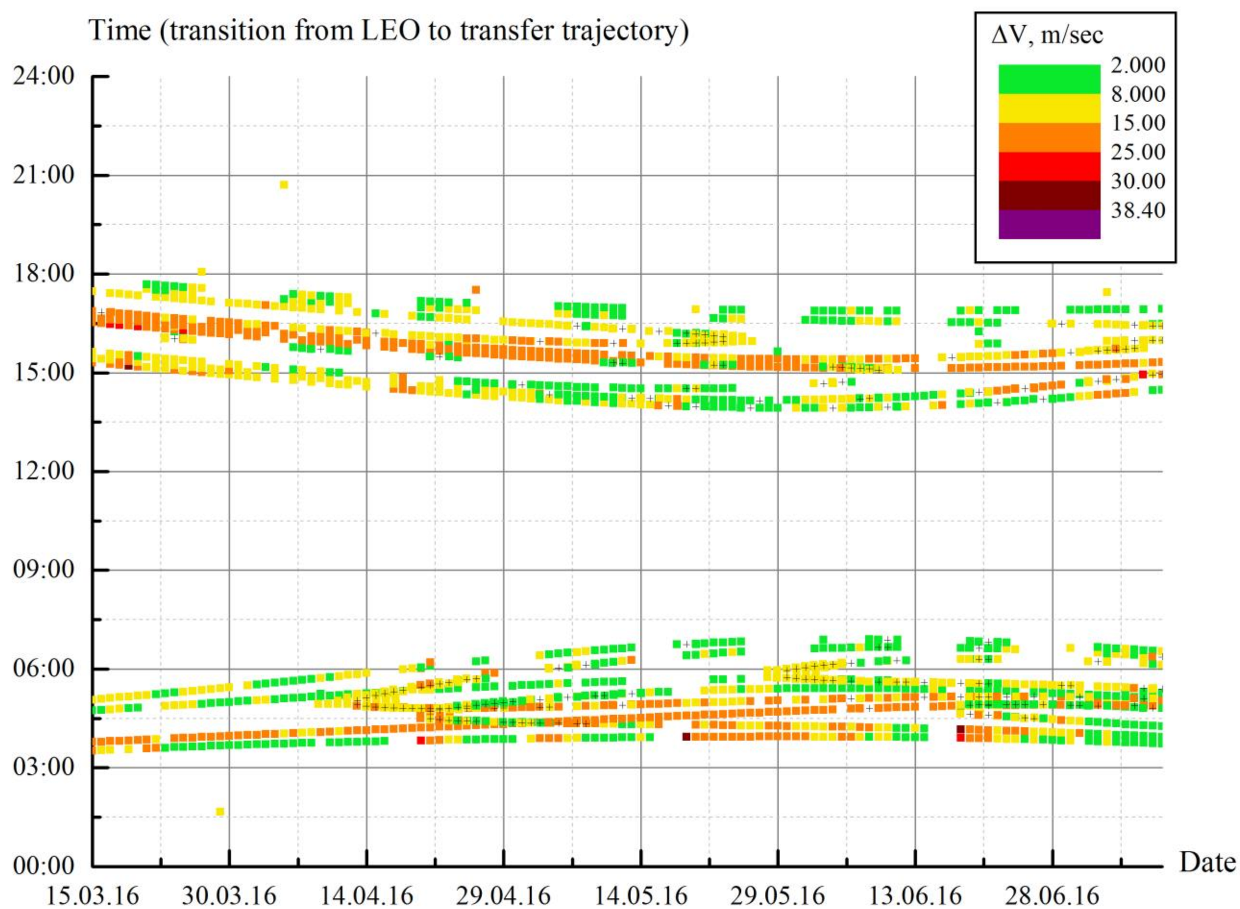

Fig 5: "Spectr-RG" launch windows from 15.03.2016 till 18.07.2016. Colour marks stationkeeping ΔV costs.

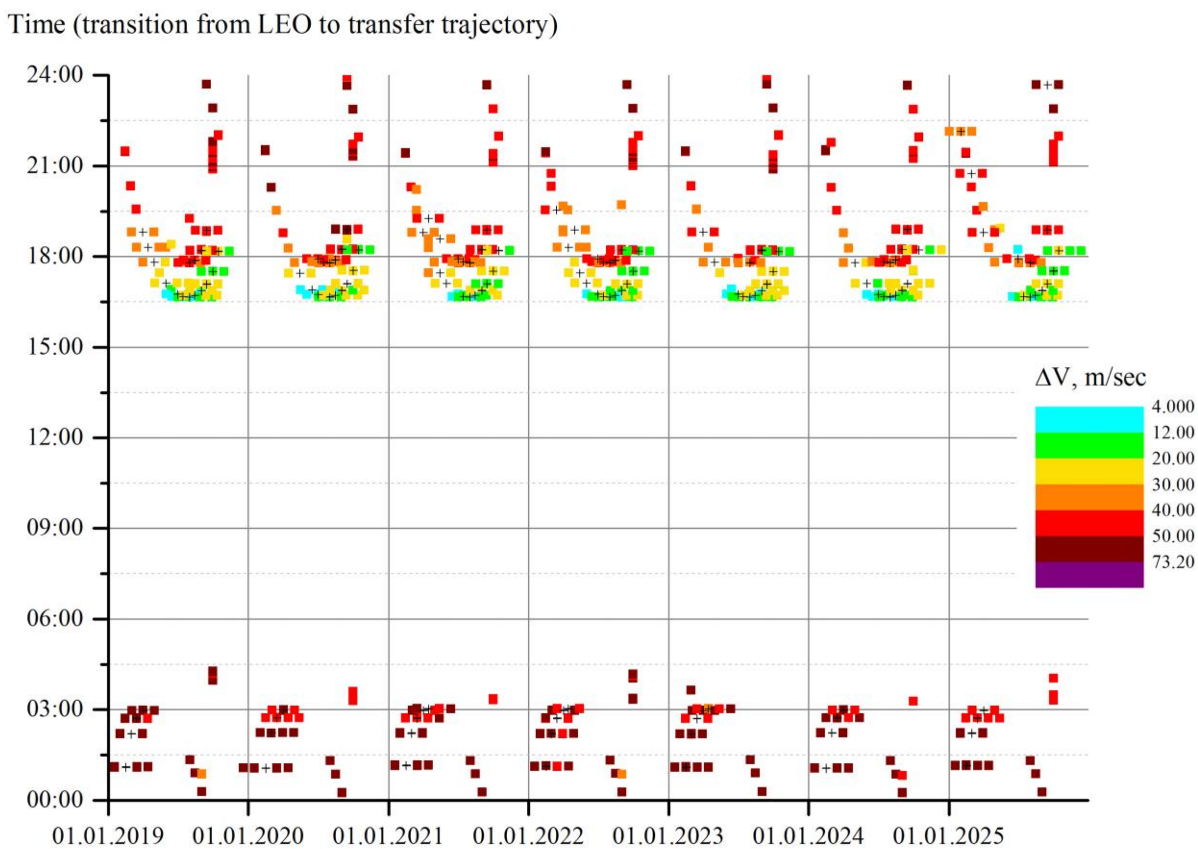

Fig 6. “Millimetron” launch windows from 15.02.2019 till 15.12.2025. Colour marks stationkeeping ΔV costs.

15˚m/sec nearly for every launch date. Nominal trajectories for “Spectr-RG” and “Millimetron” missions have been obtained by performing the calculation described above in the full Solar system ballistic model. All the restrictions such as Earth and Moon shadow avoidance conditions and constant radio visibility from the Northern hemisphere have been met.

VII. REFERENCES

[1] Gómez G., Llibre J., Martínez R., Simó C. Dynamics and Mission Design near Libration Points. - Volume 1. Fundamentals: The Case of Collinear Libration Points. World Scientific, Singapore. 2001.

[2] Ilin I.S., Sazonov V.V., Tuchin A.G. Construction of the local orbits near the L_2 libration point of the Sun – Earth system. Cosmic Research, vol. 3, 2014.

[3] Richardson D.L. Analytic construction of periodic orbits about the collinear points. Celestial Mechanics, vol. 22, P. 241-253. Oct 1980.

[4] Gurfil P., Meltzer D. Stationkeeping on Unstable Orbits: Generalization to the Elliptic Restricted Three-Body Problem. The Journal of Astronautical Sciences, vol.54, №1, January-March 2006, pp. 29-51.

[5] Lidov M.L., Lyakhova V.A., Teslenko N.M. One impulse flight to the quasi periodic orbit in the vicinity of the Sun – Earth system L_2 point and some related common problems. // Cosmic research. 1987. T. XXV. № 2. P. 163–185.

[6] Lidov M.L., Lyakhova V.A., Teslenko N.M. The trajectories of the Earth –Moon – halo orbit in the vicinity of the Sun – Earth system L_2 point flight // Cosmic research. 1992. vol. 30. № 4. P. 435–454.

[7] Ilin I.S., Zaslavskiy G.S., Lavrenov S.M., Sazonov V.V., Stepanyants V.A., Tuchin A.G., Tuchin D.A., Yaroshevsky V.S. Halo orbits in the vicinity of the Sun – Earth system L_2 point and the transfer to such orbits. Cosmic Research, to be published.

[8] Ilin I.S., Sazonov V.V., Tuchin A.G. Construction of the flights from low Earth orbits to local orbits near the libration point in the Sun – Earth system. Cosmic Research, vol. 3, 2014.

[9] Ilin I.S., Zaslavsky G.S., Lavrenov S.M., Sazonov V.V., Stepanyants V.A., Tuchin A.G., Tuchin D.A., Yaroshevky V.S. Ballistic design of transfer trajectories from LEO to halo orbits in Sun-Earth L_2 point vicinity. Cosmic Research, vol. 6, 2014.